What happens to Liquid Helium 3 at very low Temperatures?

By E. R. Dobbs, London*)

At room temperature helium 3 is a rather uninteresting, very expensive, inert gas with essentially the same properties as its well-known, comparatively cheap isotope, helium 4. It is expensive because it is produced by the beta decay of tritium gas, a byproduct of nuclear reactors. Helium 3 gas contains the simplest of all molecules and behaves as an almost perfect gas down to its critical temperature 3.3 K. A low temperature physicist who studied the properties of ^{3}He down to the lowest temperature obtainable with liquid ^{4}He (about 0.8 K) and concluded that it was extremely dull compared with the exciting, superfluid ^{4}He isotope would not have been contradicted by many theorists before the 1960's. The discovery of not one, but at least two, new superfluid phases of ^{3}He in 1972 transformed very low temperature physics. Below 2.6 mK a wealth of fascinating phenomena exist, which were unknown four years ago.

In this article I discuss the changes in the properties of condensed ^{3}He as the temperature is lowered. There are three regions: first, between 3.3 and 0.8 K, where it is a simple quantum fluid; secondly, between 0.8 K and 2.6 mK, where it is a normal Fermi liquid; finally, below 2.6 mK, where it forms the A and B superfluid phases.

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Quantum Fluid

Helium 3 atoms, like the other inert gas atoms, are well represented by the spherically symmetrical Lennard-Jones pair potential (fig. 1). The two characteristic parameters — σ, the hard core diameter and ε the binding energy for a pair at 0 K — are known from the properties of the gas to be $\sigma = 0.256$ nm and $\varepsilon/k_B = 10.2$ K. The van der Waals binding energy is extremely weak, but would classically produce a close-packed crystal of about 12 cm^3 mol^{-1}. Such a crystal can only be made near 0 K by compressing ^{3}He with about 1400 bar! On releasing the pressure the solid expands until at a molar volume of 26 cm^3 and pressure of 34 b it melts to

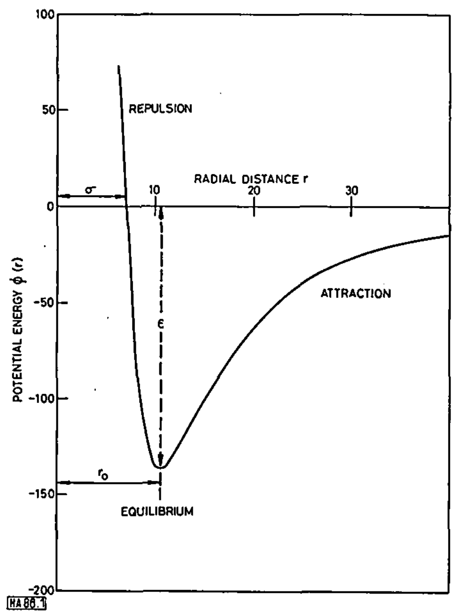

Fig. 1. Lennard-Jones pair potential for inert gas atoms: σ = hard core diameter, r_0 = classical equilibrium separation at 0 K, ε = binding energy for a pair.

form a liquid whose molar volume is 37 cm^3 — about three times the classical volume — at zero pressure.

The huge expansion of solid helium was shown by *de Boer* (1948) to be due to the quantum nature of the solid. He expressed the molar volume V_0 of condensed gases in reduced units $V_0^* = V_0/N_0\sigma^3$ and found an empirical relation between V_0^* and a reduced de Broglie wavelength $\Lambda^* = \Lambda/\sigma = h/\sigma m^{1/2}\varepsilon^{1/2}$, where Λ is the de Broglie wavelength and m the mass of the atom. Using the Leonard-Jones potential for σ, ε he successfully predicted (fig. 2) that solid ^3He would have a molar volume of 27 cm^3. Thus solid ^3He is less than half as dense as it would be, if its atoms were not in vigorous zero-point motion owing to their very small mass, and is a quantum crystal.

The relative energies of the condensed states of ^3He are shown in fig. 3, where U_0 is the close packed lattice energy from the Lennard-Jones potential, E_z is the zero point vibrational energy measured by *Dugdale* and *Franck* (1964) and $E_0 = U_0 + E_z$ shows the shallow minimum at the molar volume (26 cm^3) of the solid. *London* (1954) has shown that when the potential energy for a loose, liquid-like structure is added to the zero point kinetic energy appropriate to a particle trapped in a hollow cavity due to its neighbours, the resultant internal energy

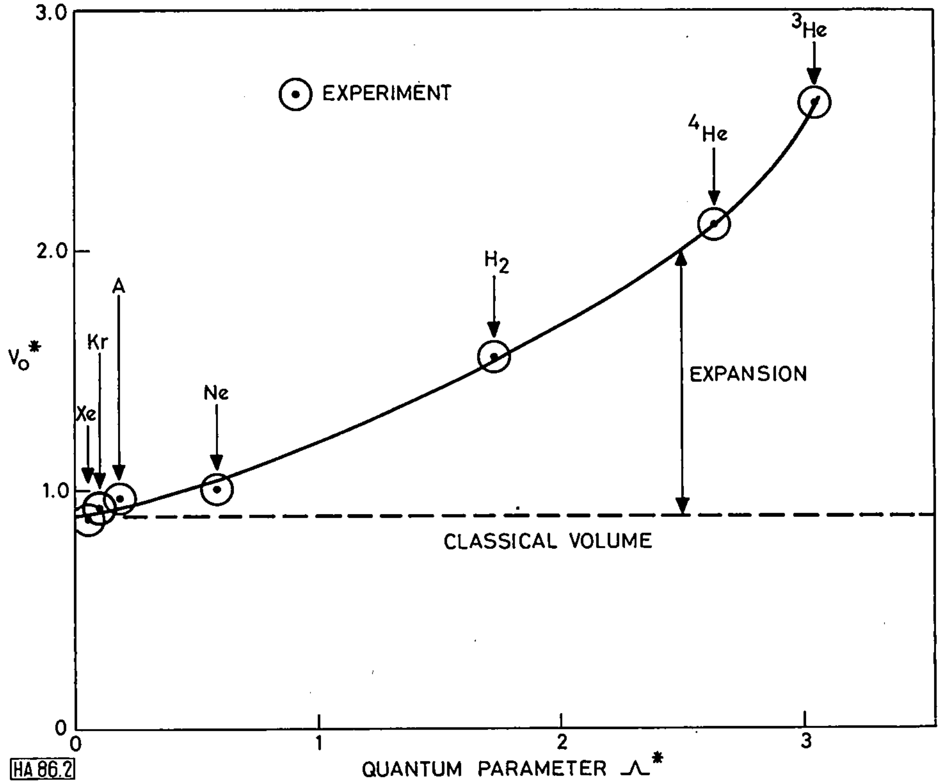

Fig. 2. Molar volumes of crystals plotted in de Boer's reduced units (V_0^*) against a reduced de Broglie wavelength (Λ^*) to show the quantum expansion of the lighter inert gas solids compared with the classical molar volume of the heavier inert gas crystals.

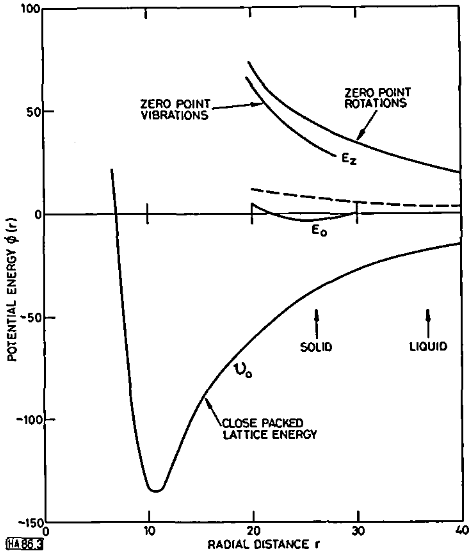

Fig. 3. Energies of condensed states of ^3He: U_0 close packed lattice energy of Lennard-Jones atoms, E_0 = internal energy of crystal with lattice energy U_0 plus zero point energy E_z, — internal energy in liquid (see text).

(dashed line in fig. 3) has no minimum, but a small positive energy at the molar volume (37 cm^3) of the liquid. The ^3He atoms in this quantum liquid have so much freedom to librate, and their interatomic forces are so weak, that they behave above 0.8 K as a Fermi gas.

Since ^3He is a fermion and ^4He a boson, the liquids helium provide a unique verification of the properties of the Fermi and Bose gases at very low temperatures. The heat capacity, for example, of a Bose gas rises at low temperatures (fig. 4) until it reaches the temperature (T_c) of the Bose-Einstein condensation, when it falls rapidly to zero. On the other hand a Fermi gas condenses slowly and continuously into its ground state, the Fermi sphere of radius kT_F in momentum space, where $T_F = (h^2/8mk)(3N_0/\pi V_0)^{2/3}$ is the Fermi temperature. Below T_F the heat capacity of a Fermi gas falls monotonically to zero (fig. 4), becoming linear in the degenerate region for $T \ll T_F$.

As is well known, liquid ^4He shows the striking λ-point transition from normal liquid He-I to the ordered liquid He-II, which becomes a perfect superfluid at $T \ll T_c$. The λ-point is a logarithmic infinity and its enormous size shows that the ^4He condensate is formed from interacting Bose particles, that is superfluid ^4He is a Bose liquid condensate. For liquid ^3He properties such as its heat capacity (C_V) and its magnetic susceptibility (χ_m) behave qualitatively like those of a Fermi gas. Thus C_V is linear in T below 50 mK and χ_m is independent of T below 40 mK, as expected by analogy with the Pauli susceptibility of metals. However, experimentally T_F is about 1.5 K, about one-third of that calculated (4.9 K) for a Fermi gas of ^3He atoms. In this degenerate region the ^3He ground state is evidently due to interacting Fermi particles forming a Fermi liquid.

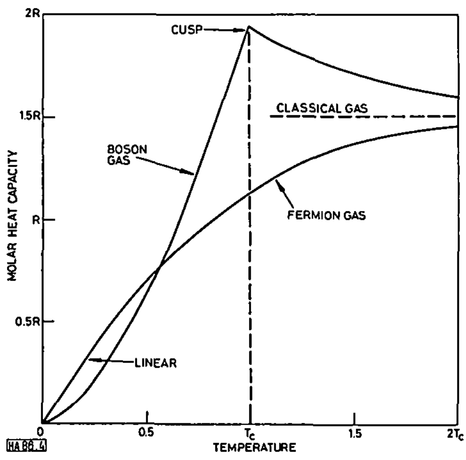

Fig. 4. Molar heat capacity of gas of fermions compared to those of boson and classical gases. The cusp is characteristic of the Bose-Einstein condensation and the linear region of the degenerate Fermi-Dirac gas.

Fermi Liquid

In *Landau's* theory[1]) the ^3He atoms are represented by the quasiparticle excitations of effective mass m^* of a degenerate Fermi liquid. The quasiparticles interact with a momentum-dependent and spin-dependent force, which Landau characterises by two series of dimensionless parameters: F_0, F_1, $F_2 \ldots$ and Z_0, $Z_1 \ldots$. Fortunately it is possible to represent the experimental properties of liquid ^3He quite well by just three parameters: F_1, F_1 and Z_0. These are found from the following relations for the speed of longitudinal sound (c_s), C_V and χ_m in the liquid (L), normalised to their values in the Fermi gas (G):

$$(c^L{}_s/c^G{}_s)^2 = (1 + F_0)\,(m/m^*),$$
$$(C^L{}_V/C^G{}_V) = (m^*/m) = (1 + F_1/3),$$
$$\chi^L{}_m/\chi^G{}_m = (1 + Z_0/4)^{-1}\,(m^*/m).$$

Rounded values of thes parameters at zero and melting pressure are given in Table 1.

In Landau theory the effective mass m^* is directly related to F_1 and so both are determined from the linear heat capacity below 40 mK. The effective mass, and hence the density of states at the Fermi surface, are enhanced three times at zero pressure and six times at the melting pressure (table 1). The velocity of sound more than doubles in this pressure range and so F_0, the compressibility parameter, increases dramatically from 10 to 94. On the other hand the spin parameter, Z_0, is almost independent of preasure, but its large negative value shows that the spin dependent exchange forces favour parallel alignment of the spins at all pressures. If the exchange forces were a little stronger ($Z_0 \leqslant -4$), then liquid ^3He would be ferromagnetic!

Table 1. Landau parameters.

P(b)	m^*/m	F_0	F_1	Z_0
0	3	10	6	—2.7
34	6	94	16	—3.0

1 **For an introduction to Landau theory see Wilks (1967).**

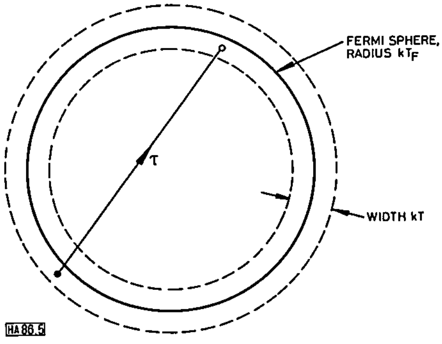

Fig. 5. The Fermi sphere in momentum space for ^3He atoms, showing the region of width kT for quasiparticle energies on Landau theory at $T = 0.15$ K, with the Fermi temperature $T_F = 1.5$ K.

The transport properties of liquid ^3He can also be explained by Landau's theory. At very low temperatures ($T < 50$ mK) the quasiparticles form a rarefied gas in momentum space (fig. 5) with energies within a band of width kT around the Fermi surface. The number of quasiparticles available to scatter a given one is proportional to kT, while the number of final states available is also proportional to kT. As a result the collision frequency is proportional to T^2 or the relaxation time τ between collisions is proportional to T^{-2}. Consequently, in contrast to the superfluid properties of ^4He below the λ-point, the viscosity of ^3He rises dramatically in the degenerate Fermi liquid region, becoming 2 mP at 30 mK and no less than 0.3 P at 3 mK — which is the same as olive oil at 40 °C! In a similar way its heat conductivity K improves at very low

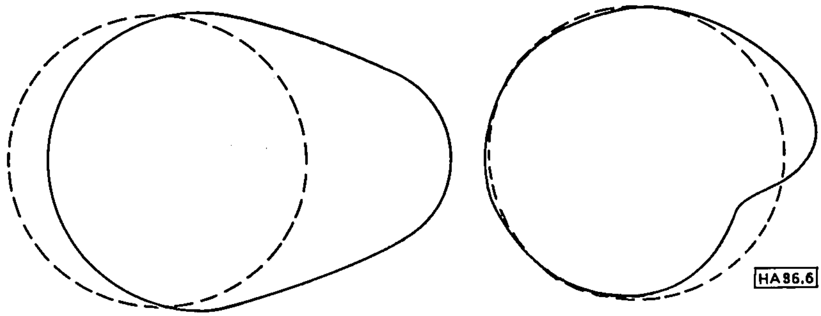

Fig. 6. The distortions of the Fermi surface of ^{3}He by zero sound modes: (left) longitudinal, (right) transverse, zero sounds; the broken curve shows the equilibrium Fermi surface.

temperatures, where $K \sim T^{-1}$, becoming the same as copper at about 3 mK.

Clearly a highly viscous liquid with a high thermal conductivity will attenuate sound waves very effectively and liquid ^{3}He should therefore become opaque to ultrasonics at $T < 50$ mK. However Landau predicted that at these low temperatures, for ultrasonic waves having angular frequencies $\omega > \tau^{-1}$, new collisionless, collective modes called zero sound should propagate. In ordinary (longitudinal) sound the Fermi surface remains spherical, but its centre vibrates about the origin of momentum space — the motion is hydrodynamic. In longitudinal zero sound at a particular instant the Fermi surface is elongated in the forward direction of propagation and shortened in the backward direction (fig. 6 left), but half a cycle later it is lengthened in the backward direction and considerably shortened in the forward direction. The motion is a collective oscillation of the Fermi surface with greater amplitude at the forward pole. In transverse zero sound the Fermi surface distortion in the forward direction is even more unusual (fig. 6, right). The first measurements of transverse zero sound were reported earlier this year by *Roach* and *Ketterson* (1976) at 12 MHz and we are currently making similar measurements at much higher frequencies. As well as providing another confirmation of Landau's theory, transverse zero sound data should enable the next parameter, F_2, to be found.

Superfluids

Superfluid helium 4 and hundreds of superconductors have been studied extensively for many years, but until *Bardeen, Cooper* and *Schrieffer* (1957) developed their theory of superconductivity there was no microscopic theory of any superfluid. Clearly superfluid ^{3}He cannot be a single particle boson condensate like superfluid ^{4}He, but must resemble the other Fermi superfluids in having pairs of quasiparticles with correlated spins[2]). For s-electrons in metals, *Cooper* (1956) had shown that an attractive interaction between quasiparticles having opposite momenta and antiparallel spins could lead to a bound state with a lower energy than twice the Fermi energy. These Cooper pairs have no angular momentum ($l = 0$) and no spin ($s = 0$) and a coherence length ("radius") on B. C. S. theory given by $\xi_0 = \hbar v_F/\pi\Delta$, where v_F is the Fermi velocity ($\hbar k_F/m^*$) and Δ is the "energy gap" parameter. Thus 2Δ is the

2 For a comparison between Bose and Fermi superfluids see Vinen (1969).

minimum energy required to excite a pair of quasiparticles from the superconducting ground state. Typically Cooper pairs are coherent for hundreds or thousands of interatomic spacings, r_0, ($\xi_0/r_0 = 140$ for Nb, 5600 for Al) in metals.

The attractive interaction between the electrons which leads to the superconducting state at sufficiently low temperatures can be visualized in terms of the polarization of the positively charged lattice of heavy ions. The passage of an electron through the lattice would leave in its wake a positively charged track, which at sufficiently low temperatures ($T \ll \Theta_D$, the Debye temperature) would revert slowly to its original, unpolarized state. Consequently a second electron entering this partially polarized region would be attracted to the positively charged track and hence effectively to the first electron. In practice such interactions produce a superconducting transition at $T_c/\Theta_D \sim 0.1$ for strong coupling superconductors like Pb and ~ 0.003 for weakly coupled superconductors like Al.

The interaction between ^{3}He atoms in liquid ^{3}He which can lead to superfluidity is more subtle. The ^{3}He atoms are inert and uncharged; there is no ion lattice to be polarized. The polarization takes place in the liquid itself, which at sufficiently low temperatures ($T \ll T_F$) is a Landau Fermi liquid. For the spin polarization, which we have seen is strong in liquid ^{3}He ($Z_0 = -3$), we can visualize a ^{3}He atom travelling through the liquid leaving in its wake a partially spinpolarized track. This polarization fades slowly and so a second ^{3}He atom coming near this track would be either attracted or repelled (depending on its spin) and so effectively interact with a spin-dependent interaction with the first ^{3}He atom. This mechanism can lead to correlated pairs of parallel-spin particles and so favours spin triplet pairing ($S = 1$) and not the spin singlet pairing ($S = 0$) of the electrons.

A similar argument can be applied to the density fluctuations associated with the compressibility parameter F_0. This leads to an attractive interaction, especially at lower pressures, which is spin-independent. By analogy with weakly coupled superconductors, we might expect a transition to superfluidity at $T_c/T_F \sim 0.003$ or about 5 mK for liquid ^{3}He. But predictions of superconducting transition temperatures are notoriously difficult in some cases (e. g. T_c for Cu has been estimated to be 15 mK, but it has been cooled below 10 mK and found to be still normal). It was therefore a relief to many theorists and a surprise to many experimenters when *Osheroff, Richardson* and *Lee* (1972) found two new phases in ^{3}He at about 2 mK at the melting pressure.

The melting curve of ^{3}He is anomalous below 0.3 K, where dp/dT is negative and so the entropy of the solid is actually greater than that of the liquid. This enables a mixture of liquid and solid to be cooled by applying pressure, a

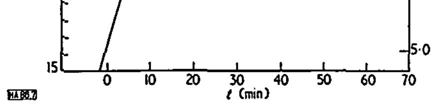

Fig. 7. The pressure-time curve for ^{3}He in a Pomeranchuk cooling cell, with the approximate equivalent temperatures on the right hand ordinate scale. The transitions are on cooling at A and B and on warming at B^1 and A^1 (Osheroff, Richardson and Lee, 1972).

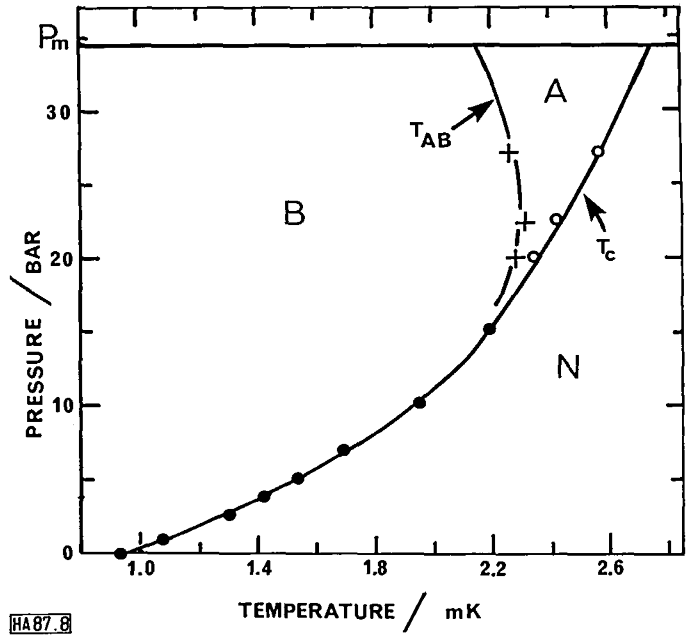

Fig. 8. Phase diagram of liquid ^{3}He in a magnetic field of 32 mT; N = normal Fermi liquid, A = superfluid A phase, B = superfluid B phase, P_m = melting pressure of solid ^{3}He (Ahonen, Haikala, Krusius and Lounasmaa, 1974).

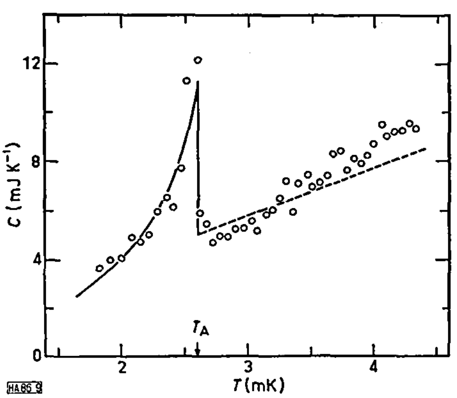

Fig. 9. Heat capacity of ^{3}He in A phase (below T_A) and in Fermi liquid (above T_A). (Anufriev, Alvesalo, Collan, Opheim and Wennerström, 1973).

technique known as Pomeranchuk cooling. Osheroff et al. observed two „kinks" in the pressure-time curve of their Pomeranchuk cell (fig. 7). As the pressure was increased the first kink appeared at A and the second at B; on releasing the pressure similar kinks appeared at B^1 and A^1 on warming. These observations have led to rapid progress in the last four years in our understanding of superfluid helium 3.

The phase diagram (fig. 8) at low magnetic fields shows the existence of two distinct superfluid phases (A and B), but no evidence of any λ-point transitions. The line T_c is a line of second order transitions of the Ehrenfest type: there is a discontinuity, but not a divergence, in the heat capacity (fig. 9), just as in the case of BCS superconductors. The line T_{AB}, on the other hand, is a line of first order transitions, terminating at zero magnetic field on the T_c line at the polycritical point with pressure P_c at about 2.4 mK. Even the small field of 32 mT has therefore a large effect,

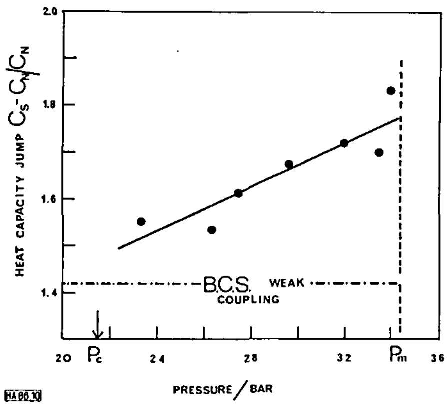

Fig. 10. Heat capacity jump $(C_S\text{-}C_N)/C_N$ at T_c for A phase — Fermi Liquid second order transition; P_m = melting pressure, P_c = polycritical point pressure (after Wheatley, 1975).

moving the apparent T_{AB}/T_C intersection down to about 2.1 mK.

A weakly coupled superconductor has a heat capacity jump, $(C_S—C_N)/C_N$, at T_c of 1.42 and evidence of the stronger coupling in superfluid ^{3}He-A is shown in fig. 10. The jump is largest near P_m, where F_0 and Z_0 are large, but falls towards the BCS value near P_c.

Further evidence that the ^{3}He superfluids resemble the BCS states of a Fermi liquid is shown in fig. 11. Here the attenuation of longitudinal zero sound is seen to increase dramatically just below T_c in ^{3}He-A, before falling to a low value, while the velocity of zero sound drops sharply below T_c to a much lower value. The drop in velocity corresponds to a rapid increase in the pairing of the quasiparticles and the peak in the attenuation is due to both pair breaking and the excitation of a collective mode of the superfluid.

These results are convincing proof of a superfluid, macroscopic quantum state in ^{3}He-A and nuclear magnetic resonance experiments have shown that the state must be a spin triplet state. In particular, *Osheroff* and *Brinkman* (1974) have shown from longitudinal NMR measurements that the A liquid is an anisotropic superfluid of the type first proposed by *Anderson* and *Morel* (1961) and later improved by *Anderson* and *Brinkman* (1973), who showed how it could be stabilized by spin fluctuation exchange. The A liquid is now known as the Anderson-Brinkman-Morel, or ABM, axial state, since it consists of only the two sub-states $S_z = +1$, $|\uparrow\uparrow>$ and $S_z = -1$, $|\downarrow\downarrow>$ and so has the same magnetic susceptibility as the normal Fermi liquid.

Less is known about the B phase, but in fig. 12 the NMR measurements of *Osheroff* (1974) show clearly that it is of the isotropic *Balian* and *Werthamer* (1963), or BW, type. In this case all three sub-states of the spin triplet exist: $S_z = \pm 1$ and the nonmagnetic: $S_z = 0$, $(1/2)^{1/2}\,|\uparrow\downarrow + \downarrow\uparrow>$ sub-state, so that $\chi_m{}^S/\chi_m{}^N$ should fall to 2/3 as $T/T_c \rightarrow 0$. This is quite different from the ABM

Fig. 11 see page 599!

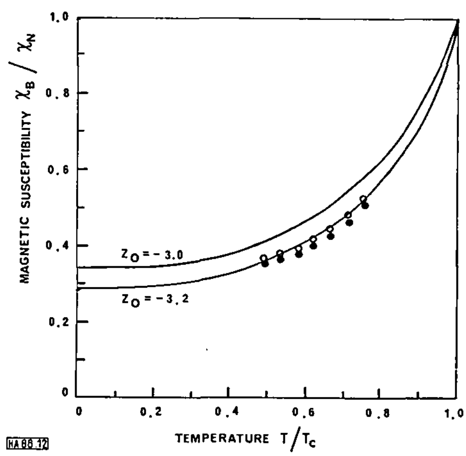

Fig. 12. Temperature dependence of the normalised magnetic susceptibility of the B phase in the high field limit of transverse NMR at P_m; o, • experimental points, curves for Balian-Werthamer models, as enhanced by Leggett (after Osheroff, 1974).

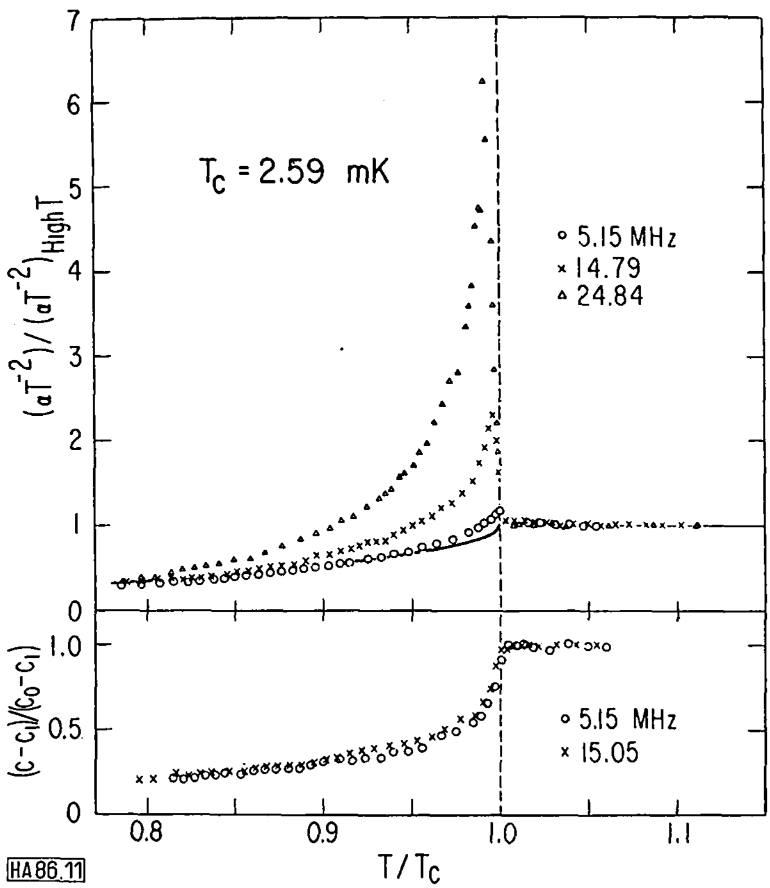

Fig. 11. Ultrasonic attenuation (α) and velocity (c) in reduced units in the A phase (below T_c) and in Fermi liquid (above T_c). (Paulson, Johnson and Wheatley, 1973).

state, where $\chi_m{}^S = \chi_m{}^N$, and the BCS spin singlet state where $\chi_m{}^S/\chi_m{}^N$ falls to zero at $T/T_c \simeq 0.25$. However, *Leggett* (1965) pointed out that the BW state was modified by the Landau spin parameter Z_0 and that for $Z_0 = -3$, $\chi_m{}^S/\chi_m{}^N = 0.35$ at $T = 0$. This is very close to the 0.28 obtained by Osheroff by a double extrapolation from his NMR data.

The identification of the A and B phases with the ABM and BW states is discussed at length by *Leggett* (1975) and *Wheatley* (1975). But this is only the beginning of the story of the superfluid phases of helium 3. Fundamentally they resemble the ^{4}He and electron superfluids in forming macroscopic quantum states. Just as the phase coherence of the single particle Bose superfluid leads to the quantum of circulation $\oint \vec{v}_s \cdot d\vec{s} = nh/m_4$, and the paired BCS superfluids has quantised flux $\Phi = \oint \vec{A} \cdot d\vec{s} = nh/2e$, so in ^{3}He the quantum of circulation must be $nh/2m_3$. But unlike the Bose superfluid, the ^{3}He superfluids do not have negligible viscosity. The viscosity of the A phase at high pressure falls sharply near T_c, but then remains at about 0.4 η_N. However, critical velocities for the onset of turbulence in ^{3}He-A are 0.02 — 0.08 cm s^{-1} and in ^{3}He — B are 0.4 — 0.5 cm s^{-1} and these are similar to those in ^{4}He in bulk (0.02 cm s^{-1}) and in confined geometries. There are many extraordinary dynamical properties of the ^{3}He superfluids depending on the temperature, magnetic field and geometry of the sample cell. For example, the healing length R_c (distance from the wall to bulk behaviour) can be ~ cm in ^{3}He-B for small magnetic fields and $T \ll T_c$! When $R_c \gg \xi_0$ the superfluids form textures like the nematic liquid crystals and complex anisotropies are revealed. Experimenters and theorists will find the ^{3}He superfluids a fascinating field of physics for many years to come.

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Bessere Chancen für Elektrofahrzeuge?

Auf dem 4. Internationalen Elektrofahrzeug-Symposium, das kürzlich in Düsseldorf stattfand, hat die VARTA AG, Hannover, eine Batterie-Neuentwicklung vorgestellt, die die Chancen für das Elektro-Straßenfahrzeug verbessern könnte. Das neue Stromspeichersystem basiert auf den Elementen Eisen und Nickel. Die vorgestellten ersten Prototypen haben eine Energiedichte, die bei 48 Wh/kg liegt; das entspricht einer Steigerung um 20 % gegenüber den Bestwerten bei Bleiakkumulatoren. Ziel der weiteren Entwicklung ist eine Energiedichte von 65 bis 70 Wh/kg.

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